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RESEARCH PAPER

Comparison of Empirical and Rock Physics Models in Estimating Shear Wave Velocity

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ABSTRACT

The measurement of shear wave velocity (Vs) using the dipole sonic imager (DSI) logging tool is regarded as a crucial physical parameter for rocks. However, not all wells have access to this data, making it essential to accurately and reliably estimate this parameter with minimal uncertainty to determine reservoir characteristics effectively. The Vs estimation approach in this study included empirical methods and two rock physic models. In empirical methods, empirical correlations reported in studies are used. In the second approach, two rock physics models, Gaussmann and Xu-Payne, which are more complicated than the experimental models, were chosen to determine the characteristics of the Vs. The main innovation of this paper is the comparison of all the mentioned methods in Vs estimation. Correlation coefficient (R2) and Average Relative Error (ARE) were chosen as statistical comparison criteria. Based on the final findings, the Greenberg and Castagna method, incorporating Gaussmann fluid replacement theory, exhibited consistent performance and improved estimation accuracy of Vs with R2 and ARE value of 0.9067 and 3.2292, respectively. The suggested approach here has the potential to be employed in various other oil and gas exploration fields and can provide accurate Vs estimates.

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1. INTRODUCTION

Shear wave velocity (V_s) has extensive applications in geophysical and petrophysical investigations, particularly for the computation of geomechanical parameters[1, 2]. In order to yield valuable insights into reservoir characteristics, its petrophysical evaluation is necessary [3]. In addition, understanding the interaction between this parameter with reservoir formation lithology and pore pressure is of great importance [4-6]. Typically, pressure wave velocity (P-wave) is derived from sound transit time measurements, whereas V_s is obtained either through dipole sonic imager (DSI) well logs or laboratory analysis of core samples. In fact, since the laboratory measurement of V_s is a core non-destructive test, these tests are performed before the enhanced oil recovery related tests such as flooding, wettability,

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etc.[7-9]. However, the availability of core samples and DSI well logs is restricted to a limited number of wells within a field, whereas P-wave transit time measurements are accessible in most wells[2, 10]. Considering this, rock physics models and statistical empirical regression methods have been developed by researchers such as Pickett (1963)[11], Carroll (1969)[12], Eskandary and others (2003)[13], and Brocher (2005)[14], etc. In this study, these methods were presented and compared in order to identify the best one in the studied reservoir.

Utilizing V_s is instrumental in the development and formulation of a mechanical earth model (MEM), which serves as a primary objective in geomechanical investigations. The mechanical earth model involves numerically determining the mechanical properties of rocks and their stress regime[15-17]. Empirical regressions have been suggested by numerous researchers as a means to estimate V_s. It is noteworthy that in most of the studies, limited parameters have been used to estimate the V_s. It is also possible that the relationships presented for a specific type of lithology or for a specific geological area are expressed and cannot be generalized to other types of rocks or other area. The impact of rock properties such as grain size and pore volume on shear velocity estimation in various rock types have been explored and highlighted as crucial parameters in determining the velocity of shear and compression waves[18]. Similarly, In their research, Miller and Stewart examined the role of clay minerals in estimating shear velocity in rocks[19]. In his analysis, Wantland made an assumption regarding the Poisson's ratio for reservoir rocks to estimate shear wave velocities. However, it is important to note that Poisson's ratio exhibits significant variation in practical scenarios, raising doubts about the accuracy of the estimated shear data.

The development of rock physics models and V_s estimation for sandstone reservoirs has made great progress. However, in the case of carbonate reservoirs, it has not been like that due to the existence of inhomogeneities and inherent complexities[20]. In addition to using the stated models, Xu and White employed a combination of the Gaussmann model and the Kuster and Toksoz models to predict V_s [21]. Their approach relied on incorporating porosity and clay mineralogy as key factors in the prediction process. Xu and White based on the combination of Gaussmann and Kuster and Toksoz models [22] predicted V_s using porosity and clay mineralogy. They show that the rock physics models for shaly sandstones can be used for carbonate rocks with some limitations. Of course, Xu and Payne were able to develop a new model for carbonate rocks in addition to shaly sandstones by presenting a rock physics model[23]. Based on Gaussman's theory, it is assumed that the pore pressures caused by the wave are balanced throughout the pore space. However, this assumption may not hold true for high-frequency acoustic waves, particularly in low-permeability rocks like sandy shale, carbonate, and shale [24, 25]. Various researchers have highlighted the significance of shear waves in the assessment of mechanical properties of rocks within the domains of geoscience and engineering. Moreover, in addition to lithology and pore fluid identification, these studies emphasize the crucial role played by shear waves [26-28]. It is worth mentioning that recently, the use of artificial intelligence and machine learning in estimating the V_s, along with the use of empirical methods and rock physics models, has been greatly appreciated by researchers. Since this topic is not within the scope of this article, it will not be discussed here. However, prominent articles that compare machine learning methods and conventional methods are summarized in Table 1.

In this paper, the problem of V_s estimation with imprical methods and rock physics model in a carbonate reservoir were discussed. The data used in this research are the petrophysical logs of a well from an oil field in southwest Iran. The dataset utilized in this study, extracted from log reports, comprises various parameters such as V_s and compressional wave velocity (Vp) obtained from DSI, as well as density (RHOB), neutron porosity (NPHI), gamma rays (GR), photoelectric effect (PEF), and resistance (LLD). Using these data, the performance of empirical and rock physic methods in V_s estimation was investigated. The empirical methods used in this stydy were Pickett, Carroll, Castagna, Greenberg and Castagna, Eskandari, Brocher, Han, Miller-Stewart, Liu and Chen and Vernik. The rock physic methods included Gassmann and Xu and Payne methods. Finally, by comparing the results, using statistical parameters such as R-square and Average Relative Error (ARE), the best method was selected.

Table 1. Summary of some prominent studies about Vs estimation

Ref.	Main object	Conventional methods used	Output
Azadpour and others (2020) [29]	Proposing a combination of rock physics and machine learning methods in V_s estimation on a carbonate reservoir	- Gassmann equation - Xu-Payne rock pyisic model	Improved VS prediction compared to the routine Xu-Payne model
Sohail and Hawkes (2020) [25]	Evaluation of empirical and rock physics models to estimate V_s in a potential shale gas reservoir using wireline logs	- Empirical methos (such as Pickett, Castagna, Kuster and Toksoz, etc.) - Xu and White rock physic model	Rock physics model can be optimized using the measured compressional wave velocity. Biot's model improved the performance of Xu and White's model in gas-saturated sandy shale.
Zhang and others (2020) [30]	V_s prediction based on a statistical rock-physics model in combination with Bayesian inversion framework	- Xu–White model rock physic model	The new method can improve accuracy of velocity prediction and provide additional statistics of the estimation.
Gang Feng and others (2023) [31]	V_s prediction based on deep neural network and theoretical rock physics modeling	- Two empirical methos (Han and Castagna)	The prediction accuracy and generalization performance are better than those of these two common empirical formulas.
Du and others (2018) [32]	Analysis of Artificial Neural Network and Rock Physics for the Estimation of V_s in a highly Heterogeneous Reservoir	- Gassmann method	ANN provides more realistic results because of its discriminatory power and observe the actual trend in VS estimation
Vukadin (2023) [33]	V_s estimation based on rock physics modelling of a limestone gas reservoir In combination with seismic modelling	Kuster–Toksöz and Xu–Payne rock physic models	the proposed workflow gives an adequate estimation of S-wave velocities.
Seifi and others (2019) [34]	Estimation of V_s by Ordered Weighted Averaging (OWA) of Rock Physics Models in a Carbonate Reservoir	- Gassmann method - Xu–Payne rock physic model	the OWA model gives the best compatibility with the original well-log data

2. Methodology

2.1. Empirical methods

Mathematical analysis serves as a valuable tool for examining the connections between independent and dependent variables, resulting in the formulation of a set of regression equations [10]. The regression equations offer an estimated correlation between the dependent and independent variables, providing an approximation of their relationship. Regression analysis can be categorized as linear or non-linear. Linear regression involves modeling data using linear independent variables or predictor functions, and it aims to predict unknown model variables based on the available data. On the other hand, non-linear regression entails modeling data using a function that represents a non-linear combination of model parameters [35]. Among the empirical models available, the ones that show the most promise and receive widespread acceptance are presented for estimating V_s in a reservoir are introduced in this study and their results in estimating V_s of a well in one the oil fields in southwest Iran are compared.

2.1.1. Pickett method

This method was introduced by Pickett in 1963 using wire-line log data. The ratio of compression wave velocity to V_s shows certain information about lithology. However, the accuracy of this method is related to the P-wave velocity value when V_p is greater than 3 Km/s. Several researchers have used this method in their study [2, 25, 36] and the relationship is presented in Table 2.

2.1.2 Carroll Method

Carroll (1969) proposed an empirical relationship between the compressional and shear wave velocities. The correlation presented holds true for rocks with Poisson's ratio ranging from 0.22 to 0.28. By considering the elastic equations, it can be deduced that the V_p/V_s ratio is expected to fall within the range of 1.61 to 1.85. This empirical method has been used[2, 35] and the relationship in this method is presented in Table 2.

2.1.3 Castagna Method

The Castagna equation stands out as one of the extensively employed correlations for predicting V_s.Castagna (1985) introduced empirical equations to estimate V_s specifically for sandstone, limestone, shale, and dolomite rocks. This method has also been investigated in several studies [2, 25, 35, 36]. The correlation equation of this method is presented in Table 2.

Table 2. Regression and empirical models in Vs stimation

No.	Method name	Equation	Unit	Lithology	Ref.
1	Pickett	$V_s = \frac{V_p}{1.6}$	km/s	Sandstone, Dolomite, Limestone	[3]
2	Carroll	$V_s = 0.937562 V_p^{0.81846}$	kft/s	Granite and volcanic rocks	[4]
3	Castagna	$V_s = 0.862V_p - 1.172$	km/s	Mudrock	[5]
4	Greenberg-Castagna	$V_s = -0.05509{V_p}^2 + 1.0168V_p - 1.0305$	km/s	Limestone	[8]
		$V_s = 0.8042V_p - 0.8559$	km/s	Sandstone	
		$V_s = 0.583V_p - 0.07776$	km/s	Dolomite	
		$V_s = 0.77V_p - 0.8674$	km/s	Shaly limestone	
5	Eskandari	$V_s = -0.1236{V_p}^2 + 1.612V_p - 2.3057$	km/s	Limestone	[9]
6	Brocher	Vs=0.07858-1.2344Vp + 0.7949 Vp^2-0.1238 Vp^3+ 0.0064Vp^4	km/s	Sandstone, Dolomite, Limestone, Shale	[10]
7	Han	$V_s = 0.79V_p - 0.79$	km/s	Sandstone	[11]
8	Miller- Stewart	$V_s = 0.8V_p - 861$	m/s	Sandstone	[12]
		$V_s = 0.448V_p + 496$	m/s	Limestone	
9	Liu and Chen	$V_s = 0.000158{V_p}^2 - 0.632162V_p + 2153.32$	m/s	Sandstone	[29]
10	Vernik	$V_s = \sqrt{2.84e^{-3}{V_p}^4 + 0.287{V_p}^2 - 0.79}$	km/s	Sandstone, Shale	[30]

2.1.4 Greenberg-Castagna method (modified using Gaussman theory)

Greenberg and Castagna in 1992 introduced a mathematical model that utilizes linear relationships of Vp (compressional wave velocity) to estimate V_s in brine-saturated rocks. This model allows for the estimation of V_s using other petrophysical parameters. The model developed by Castagna is derived based on the assumption of 100% brine saturation. Therefore, to estimate the in situ V_s (Vs), it is crucial to incorporate fluid effect calculations.

The process begins by measuring the velocity and density under a specific initial condition, denoted by index (1). Subsequently, a series of five steps are employed to ascertain the velocity following a fluid change:

The elastic moduli are derived from the initial velocities that have been saturated with brine and it is expressed as:

$$K_{sat}^{(1)} = \rho_b^{(1)}\left({V_p^{(1)}}^2 - \frac{4}{3}{V_s^{(1)}}^2\right) \quad (1)$$

$$\mu_{sat}^{(1)} = \rho_b^{(1)} V_s^2 \quad (2)$$

Where the parameters are introduced in the symbol table at the end of paper.

Using Gaussmann relation, the new apparent modulus is calculated:

$$K_{sat}^{(2)} = \frac{x}{(1+x)} K_{min} \quad (3)$$

$$x = \frac{K_{sat}^{(1)}}{K_{min} - K_{sat}^{(1)}} - \frac{K_{fl}^{(1)}}{\emptyset\left(K_{min} - K_{fl}^{(1)}\right)} + \frac{K_{fl}^{(2)}}{\emptyset\left(K_{min} - K_{fl}^{(2)}\right)} \quad (4)$$

- According to Gaussman's theory, the shear modulus does not change and it is given as:

$$\mu_{sat}^{(2)} = \mu_{sat}^{(1)} \quad (5)$$

- The new density is calculated from the following equation:

$$\rho_b^{(2)} = \rho_b^{(1)} + \emptyset(\rho_{fl}^{(2)} - \rho_{fl}^{(1)}) \quad (6)$$

Finally, the velocities are obtained in the new conditions indicated by index (2):

$$V_p^{(2)} = \sqrt{\frac{K_{sat}^{(2)} + \frac{4}{3}\mu_{sat}^{(2)}}{\rho_b^{(2)}}} \quad (7)$$

$$V_s^{(2)} = \sqrt{\frac{\mu_{sat}^{(2)}}{\rho_b^{(2)}}} \quad \textbf{(8)}$$

2.1.5. Other common methods

Eskandari (2003) in 2003 stated that V_s estimation using log data is an important approach in seismic exploration and reservoir development. He used the pressure wave velocity from the sonic log to estimate the V_s in the carbonate reservoir. This method has been invetigated [2, 37].

In 2005, Brocher created a plot consisting of numerous wave velocity data points covering a wide range of lithologies, including unconsolidated sediments, low porosity igneous rocks, and highly compacted metamorphic rocks. This extensive analysis led to the derivation of a nonlinear equation. The derived relation is applicable for Vp values ranging from 1.5 to 8.5 km/s.Several studies have used this method in their work[2, 35].

Han and others in 1986 conducted a study to investigate the relationship between pressure wave velocity and V_s by conducting measurements on rock cores under different pressure conditions. The research specifically focused on sandstone and took into account the impact of porosity and clay content[38]. The derived equation provides a simple yet widely utilized approach in practical applications. The equation has proved satisfactory predictive accuracy and has gained considerable recognition within the field. Its effectiveness lies in its ability to capture the essential factors of porosity and clay content, which significantly contribute to the determination of V_s in sandstone formations. Despite its simplicity, this equation remains a valuable tool in various applications, offering reliable estimations and aiding in the understanding of rock properties and behavior. Several authors have studied this method[25, 39].

Miller and Stewart in 1990 analyzed waveform logs from four wells in Alberta's Medicine River field to seek for relationships between pressure wave and shear wave velocities. They concluded that V_p increases quasi-linearly with Vs in sandstone and limestone and this method has been used[40].

Geostatistics-based method that was used to predict shear rate and other rock mechanical properties, in addition to seismic data and rock mechanical properties measured from well logs has been proposed and used [25, 41].

A blending approach utilizing the Greenberg-Castagna theory has also been used[25, 40, 42]. Table 2 shows the empirical regressions discussed above, along with the lithology used and the units of each method.

2.2. Rock physics models

Alongside the experimental models and regressions discussed earlier, rock physics models play a significant role in estimating V_s, as acknowledged by numerous researchers. Below, we present two of the noteworthy models that have been used in this context.

2.2.1. Gaussmann method

In conjunction with rock density, both pressure wave and shear wave velocities exert direct influence over the seismic response of a reservoir. Gaussmann emphasized the significance of Vs (V_s) and Vp (pressure wave velocity) as fundamental rock parameters [43]. These parameters are estimated based on the rock's elastic parameters, as expressed in the following equations:

$$V_p = \sqrt{\frac{K_{sat} + \frac{4}{3}\mu_{sat}}{\rho_{sat}}} \quad (9)$$

$$V_s = \sqrt{\frac{\mu_{sat}}{\rho_{sat}}} \quad (10)$$

The apparent and shear moduli are fundamental properties of the rock that can be determined based on its composition. To assess the impact of fluid saturation on these moduli, Gaussman's equations offer a model that can be expressed as follows:

$$K_{sat} = K_{dry} + \frac{(1 - \frac{K_{dry}}{K_{mat}})^2}{\frac{\emptyset}{K_f} + \frac{1-\emptyset}{K_{mat}} - \frac{K_{dry}}{{K_{mat}}^2}} \quad (11)$$

$$\mu_{sat} = \mu_{dry} \quad (12)$$

In the aforementioned model, the assumption made is that the shear modulus of dry rocks and saturated rocks is comparable.

The matrix density of a rock can be determined by taking the apparent average of its key minerals, a value that remains consistent for both the rock's pore fluids and essential minerals. Consequently, the alteration in density between saturated and dry rocks can be calculated using the following relationships:

$$\rho_{sat} = \rho_{dry} + \rho_{fl}\emptyset \quad (13)$$

$$\rho_{dry} = \rho_{mat}(1 - \emptyset) \quad (14)$$

Empirical correlations between matrix elastic modulus and porosity have been developed by [44-46] to estimate the elastic modulus of dry rock. Krief and others introduced the following model to calculate the apparent and shear modulus of dry rock[44]:

$$K_{dry} = K_{mat}(1-\emptyset)^{m(\emptyset)} \quad (15)$$

$$\mu_{dry} = \mu_{mat}(1-\emptyset)^{m(\emptyset)} \quad (16)$$

$$m(\emptyset) = \frac{3}{1-\emptyset} \quad (17)$$

Nur reported the following relationship:

$$K_{dry} = K_{mat}(1-\frac{\emptyset}{\emptyset_c}) \quad (18)$$

$$\mu_{dry} = \mu_{mat}(1-\frac{\emptyset}{\emptyset_c}) \quad (19)$$

By making an approximation of the Krief model, Pride and his team presented the following equations for estimating the apparent and shear modulus of dry rock.

$$K_{dry} = K_{mat}\frac{1-\emptyset}{1+C_{\emptyset}} \quad (20)$$

$$\mu_{dry} = \mu_{mat}\frac{1-\emptyset}{1+\gamma C_{\emptyset}} \quad (21)$$

$$\gamma = \frac{1+2c}{1+c} \quad (22)$$

Fluid distribution in rock pores is not uniform. In a reservoir that contains three fluids phase; gas, oil and water, the modulus of the fluid in the pores is defined as follows:

$$K_f = K_{gas} \times S_{gas} + K_{oil} \times S_{oil} + K_{water} \times S_{water} \quad (23)$$

$$S_{gas} + S_{oil} + S_{water} = 1 \quad (24)$$

The apparent density of multiphase fluids (P_{fl}) in rock pores can be calculated using the following equation:

$$\rho_{fl} = (1-S_w-S_o)\rho_{gas} + (1-S_w-S_g)\rho_{oil} + S_w\rho_{water} \quad (25)$$

where, and represent the density of gas, oil and water.

2.2.2. Xu and Payne method

The rock physics model [23] for carbonate rocks is based on the velocity model [21] for clastic rocks. In this model, the volume of pores or total porosity is divided into four types of porosity related to clay minerals, interparticle pores, microcracks, and stiff pores. Total porosity is defined as equation 26:

$$\emptyset_T = \emptyset_{Clay} + \emptyset_{Crack} + \emptyset_{Interparticle} + \emptyset_{Stiff} \quad (26)$$

Porosity related to clay mineral is defined according to the Xu and White model [21] for clay and non-clay pores given as follows:

$$\emptyset_{Clay} = V_{sh}\emptyset_T \quad (27)$$

Fractures are one of the most important features of carbonate rocks which are sensitive to environmental stresses and pressures. For this purpose, the following relationship is used:

$$\emptyset_{Crack} = \emptyset_{Init}e^{-\beta\sigma_e} \quad (28)$$

where β is a constant related to the speed of P and S waves measured in the laboratory and is the initial porosity of the crack in surface conditions (is equal to zero). Figure 1 shows the diagram of Xu and Payne rock physics model [23], where all the stages of model formation can be seen schematically.

This model includes the following five steps:

1- The elastic modulus of the rock matrix is determined using the Vigot-Reus-Hill (V-R-H) model [47, 48]. This established the upper and lower limits of the elastic modulus, respectively. The elastic modulus of the rock matrix was obtained by averaging the upper limit proposed by Vigot and the lower limit put forth by Reus[49]. The upper limit of the Vigot, MV, for a mixture of N phases of matter can be calculated as:

$$M_V = \sum_{i=1}^{N} f_{min_i}M_i \quad (29)$$

In a similar way, the lower limit of the Reus, MR, of a mixture of N phases of matter can be calculated as:

$$\frac{1}{M_R} = \sum_{i=1}^{N}\frac{f_{min_i}}{M_i} \quad (30)$$

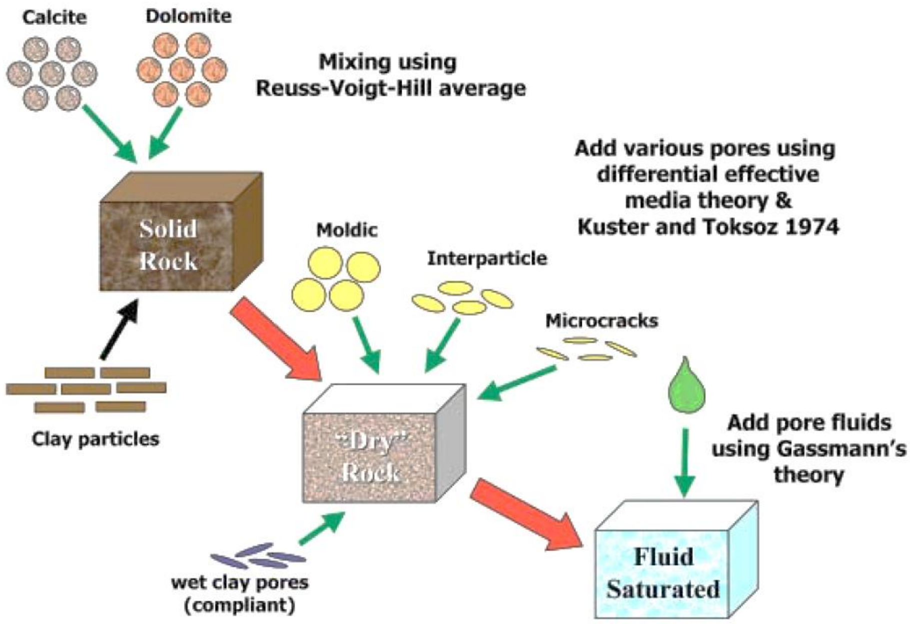


Figure 1. Diagram of Xu and Payne physics modeling steps [23].

The Vigot -Reus-Hill mean is the arithmetic mean of the Vigot -Reus limit, given as follows:

$$M_{VRH} = \frac{M_V + M_R}{2} \tag{31}$$

2- Determining the elastic modulus of dry rock is a crucial yet intricate process. It involves incorporating the appropriate pore structures into the dry rock and utilizing the isolation differential effective medium (DEM) approach to calculate the elastic modulus [50, 51]. Equations (32) and (33) are employed for the computation of the elastic modulus in dry rock.

$$K_{dry}(\emptyset) = K_0(1-\emptyset)^p \tag{32}$$

$$\mu_{dry}(\emptyset) = \mu_0(1-\emptyset)^q \tag{33}$$

Where p and q refer to coefficients that are only related to the aspect ratio (α).

3- The apparent modulus and fluid density can be determined using the Dias equation [52], which incorporates the mixing of pore fluids in the petrophysical model to calculate the apparent modulus and density of multiphase fluids. The apparent modulus can be calculated via the Reus limit, while the density can be calculated using the Vigot limit:

$$\frac{1}{K_{fl}} = \sum_{i=1}^{N} \frac{f_{fl_i}}{K_i} \tag{34}$$

$$\rho_{fl} = \sum_{i=1}^{N} f_{fl_i}\rho_i \tag{35}$$

4- Elastic modulus and saturated rock density: Gaussmann (1951) relations state how the elastic modulus of a dry or saturated rock changes if fluids are added or replaced in the rock, which is a simple but important goal. This is the fluid replacement problem which can be expressed by equations 11 to 14.

5- The P and S wave velocities of saturated rock are determined using the principles of elastic media, by considering the apparent modulus and shear modulus of the saturated rock [53], which is expressed as follows:

$$V_p = \sqrt{\frac{K_{sat}}{\rho_{sat}}} \tag{36}$$

$$V_s = \sqrt{\frac{\mu_{sat}}{\rho_{sat}}} \tag{37}$$

3 Results and Discussion

3.1 Data preprocessing and corrections

The calibration, processing, and correction of log data are crucial steps in enhancing the quality of petrophysical property data for the target reservoir, accounting for lithological and environmental effects. In this study, the available log data sets were subjected to Schlumberger environmental corrections to ensure the accurate calibration and correction.

3.2 Empirical Methods

In the previous sections, the experimental methods including Pickett, Carroll, Castagna, Greenberg Castagna, Eskandari, Brocher, Hahn, Miller and Stewart, Liu and Chen, and Vernik were introduced.

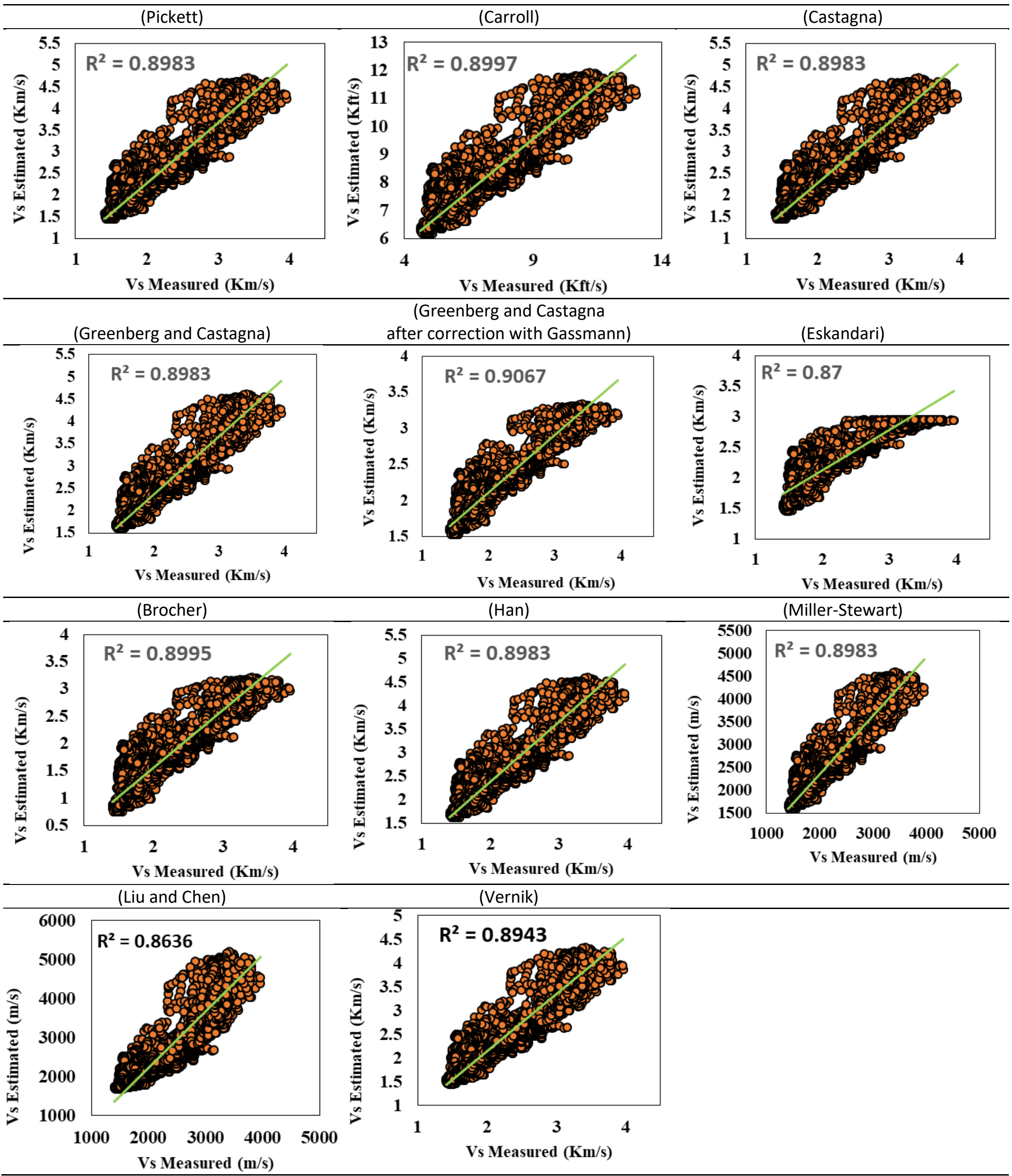


Figure 2. Estimated versus actual Vs in different methods

In order to compare different methods, the estimation results of each of the mentioned methods have been plotted versus the actual values of the V_s. Also, using statistical parameters, the accuracy of each method is reported in Table 4.

The quality of the Greenberg-Castagna method when applied in conjunction with fluid replacement (using equations 1 to 8) has been investigated. Based on the geological context, the dominant minerals found in the carbonate reservoirs of the study area are primarily calcite and dolomite. The fluid composition in this region comprises water, oil, and natural gas. Table 3 presents the elastic parameters of the minerals and fluids utilized in the prediction process. , and values were calculated using elastic limits. The value of in salt water saturation conditions was obtained after applying the fluid replacement command, and finally VS was calculated using the definition of elastic moduli.

In Figure 2, the actual V_s values are plotted against the V_s predicted by the Greenberg-Castagna method with fluid replacement. By comparing the V_s obtained from the Greenberg-Castagna Empirical method and the Greenberg-Castagna method with fluid replacement (Figure 2), it can be seen that the prediction of V_s is reasonable in both cases.

The Greenberg-Castagna method along with fluid replacement show better results. Therefore, it can be concluded that by applying the fluid replacement method, a better estimate of the shear wave can be obtained.

Table 3. Density, apparent modulus and shear modulus of minerals and fluids used to predict rock Vs

Component	Density (g/cm³)	Aparent Modulus (Gpa)	Shear Modulus (GPa)
Calcite	2.71	76.8	32
Dolomite	2.87	94.9	45
Salt water	1.1	3.2	-
Water	1	2.2	-
Oil	0.7	0.8	-
Gas	0.2	0.02	-

3.3 Rock Physic Methods

In Figure 3, the relationship between the estimated V_s values obtained from the Gaussmann model and the Xu and Payne model is compared to the DSI V_s. The results revealed a moderate correlation coefficient and limited accuracy when compared to experimental methods. It is worth noting that estimating V_s using the Gaussmann model becomes more challenging due to the significant variability in clay content, porosity, and gas saturation values within the reservoir interval.

In Xu and Payne model, the correlation coefficient shows that the accuracy of this model is better than the Gaussmann model; however, it generally cannot obtain ideal results compared to empirical methods. It is important to highlight that the Xu and Payne model introduces variations in the elastic parameters across different minerals, which can introduce uncertainties in the prediction process.

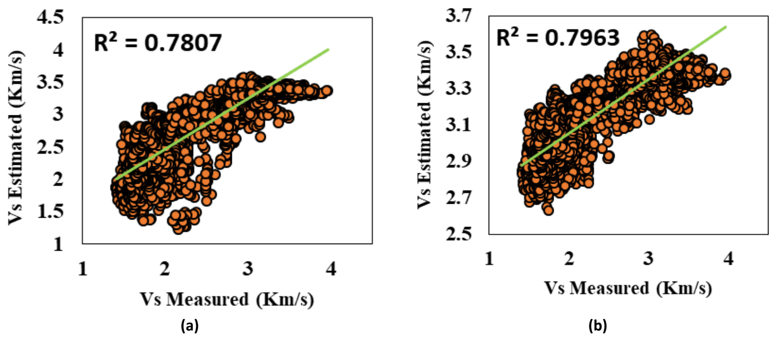


Figure 3. Estimated versus actual Vs in a) Gausmann model and b) Xu and Payne model

3.4 Statistical Analysis of Accuracy Methods

The evaluation of accuracy and reliability of the models was conducted using the statistical criteria. The mathematical formulation of the statistical evaluation parameters employed in this study is presented as follows [4]:

Correlation coefficient:

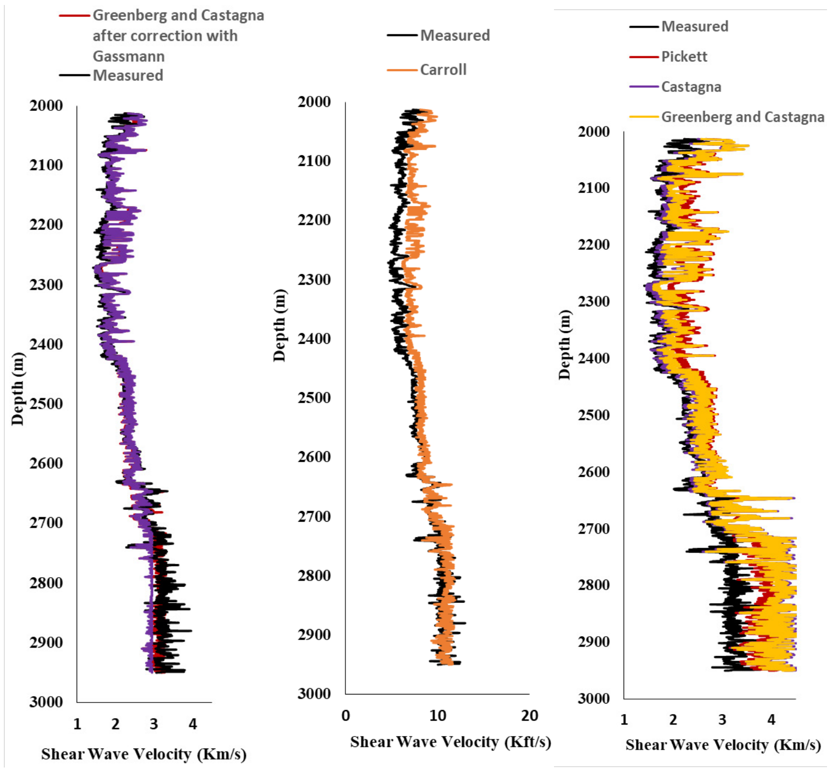
$$R^2 = 1 - \frac{\sum_{i=1}^{N}(Vs_i^{Pred} - Vs_i^{exp})^2}{\sum_{i=1}^{N}(Vs_i^{Pred} - average(Vs_i^{exp}))^2} \tag{38}$$

Average relative error:

$$ARE\% = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{Vs_i^{Pred} - Vs_i^{exp}}{Vs_i^{exp}}\right) \tag{39}$$

Table 4. Error analysis of the used models

Method/ Acuu.	Pickett	Carroll	Castagna	Greenberg and Castagna	Greenberg and Castagna after correction with Gassmann	
R^2	0.8983	0.8997	0.8983	0.8983	0.9067	
$ARE\%$	23.0963	13.4924	16.9398	19.8059	3.2292	
	Eskandari	Brocher	Han	Miller-Stewart	Liu and Chen	Vernik
R^2	0.87	0.8995	0.8983	0.8983	0.8636	0.8943
$ARE\%$	20.6053	19.5864	19.9799	18.7487	15.0819	8.7309
	Gausmann	Xue and payne				
R^2	0.7807	0.7963				
$ARE\%$	18.7819	40.5239				



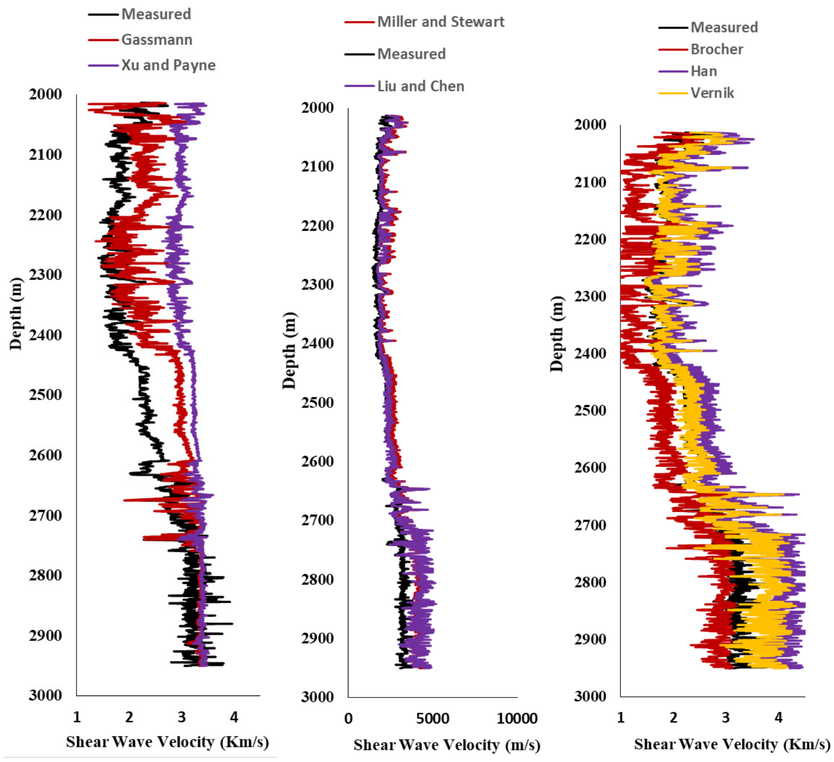


Figure 4. Estimated Vs versus depth in different methods

The error of each method is reported in Table 3. Among the used methods, the lowest error is attributed to the Greenberg-Castagna method with fluid replacement. Also, in Figure 4, the value of the estimated V_s is drawn in relation to the depth.

4 Conclusion

In this study, various empirical and rock physics methods were investigated to estimate V_s based on well report data. The methodology section provided a comprehensive review of these estimation methods and after careful evaluation, we chose an efficient and effective approach. The results showed that the empirical methods used to predict the V_s yielded more satisfactory results than the rock physics models. However, among the limitations of these methods, it is important to note that the empirical methods tend to slightly overestimate the data points of the V_s. The limitations of these versions can also be pointed to the use of only Vp parameter because it seems that the use of other petrophysical parameters increases the accuracy of estimation. Also, these methods may be developed for a specific type of lithology, and it is not possible to use some of them for all types of rocks.

Among the investigated empirical methods, the Greenberg-Castagna method, which includes fluid substitution for V_s prediction, showed superior performance compared to other empirical methods in the studied field. This method showed better accuracy and reliability in estimating V_s in the reservoir. The R-squre and ARE values for this method were obtained 0.9067 and 3.2292, respectively.

For future research, it will be useful to explore the geomechanical equations to gain insight into the dynamic behavior of the reservoir, as well as evaluate the influence of pressure and fluid conditions within the reservoir. By combining such considerations, a more comprehensive understanding of reservoir behavior can be obtained which in turn leads to improved predictions and reservoir management strategies.

Symbols

K_f	apparent modulus of pore fluid
ρ_{sat}	apparent density of saturated rock
f_{fl_i}	fraction of each fluid phase i
f_{min_i}	volume fraction of mineral component i
$\emptyset_c$	critical porosity which depends on lithology and pore structures
K_0	apparent modulus of the rock matrix
K_{dry}	apparent modulus of dry rock
K_{fl}	apparent modulus of the fluid
K_{gas}	apparent modulus of gas
K_i	apparent modulus of each fluid phase i
K_{mat}	apparent modulus of rock matrix
K_{min}	apparent modulus of the mineral matrix
K_{oil}	apparent modulus of oil
K_{sat}	apparent modulus of saturated rock
K_{water}	apparent modulus of water
M_i	elastic modulus of mineral component i
S_{gas}	saturation of gas
S_{oil}	saturation of oil
S_{water}	saturation of water
V_p	pressure wave velocity
V_s	Vs
V_{sh}	shale volume
μ_0	shear modulus of the rock matrix
μ_{dry}	shear modulus of dry rocks
μ_{mat}	shear modulus of rock matrix
μ_{sat}	shear modulus of saturated rock
ρ_b	density
ρ_{dry}	density of dry rock
ρ_{fl}	density of the fluid
ρ_i	density of each fluid phase i
ρ_{mat}	density of rock matrix
$\emptyset$	porosity
c	consolidation coefficient that is related to the pore structure

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مقاله تحقیقاتی

مقایسه مدل های تجربی و فیزیک سنگ در تخمین سرعت موج برشی

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چکیده

اندازه گیری سرعت موج برشی با استفاده از ابزار دوقطبی صوتی (ISD) به عنوان یک پارامتر فیزیکی حیاتی برای سنگ ها در نظر گرفته می‌شود. با این حال، همه چاه ها به این داده‌ها دسترسی ندارند، بنابراین برآورد دقیق و قابل اعتماد پارامتر سرعت موج با حداقل عدم قطعیت برای تعیین ویژگی های مخزن به طور موثر ضروری است. رویکرد پیش‌بینی سرعت موج برشی در این مطالعه شامل روش‌های تجربی و روش مدل فیزیک سنگ است. در روش های تجربی از همبستگی های تجربی گزارش شده در مطالعات استفاده شده است. در روش دوم، دو مدل فیزیک سنگ گاسمن و شو و پاین که پیچیدگی بیشتری نسبت به مدل‌های تجربی دارند، برای تعیین سرعت موج برشی انتخاب شده‌اند. در این تحقیق مقایسه تمامی روش های ذکر شده بر اساس ضریب همبستگی (۲R) و میانگین خطای نسبی (ERA) است. بر اساس یافته‌های نهایی، روش گرینبرگ و کاستانیا، با ترکیب نظریه جایگزینی سیال گاسمن، عملکرد بهتری داشته و دقت پیش‌بینی سرعت موج برشی را با مقدار ۲R = ۰/۹۰۶۷ بهبود بخشید..

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