A Comprehensive Solution for Partially Penetrating Wells with Various Reservoir Structures

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ARTICLE INFO

Article history:
Received: May 28, 2016
Accepted: November 7, 2016

Keywords:
Partially penetrating well
Pressure transient analysis
Newman's product method
Fully penetrating well
Laplace transform domain

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ABSTRACT

This paper presents analytical solutions and simulations for pressure transient behavior of the partially penetrating wells (PPWs). The Newman's product method was adopted to develop the basic instantaneous source functions which characterize the response of PPWs. These results were obtained based on the solution of fully penetrating wells (FPWs) and they were presented in Laplace domain. Further, several synthetic examples were provided and simulated to investigate the responses for a PPW and to analyze the effects of different parameters which support the obtained theoretical results.

1. Introduction

The pressure behavior of partially penetrating wells (PPWs) has been investigated extensively in the literature [1-5]. Many scholars have considered flow of the fluid in the reservoir toward the well as a radial flow with an additional pressure drop near the wellbore caused by the partial penetration [6-11]. This additional pressure drop is characterized by a positive geometrical skin that is called pseudoskin, due to partial completion. Several analytical and empirical dispositions have been developed to evaluate the pseudoskin and the time for which the assumption of radial flow becomes valid [9-13]. On the basis of the spherical flow, some researchers have provided methods for interpreting pressure transient well test data [14-17]. Many solutions have been developed to the two dimensional (2D) diffusivity equation, which included flow of the fluid in the vertical direction [14, 18-22].

On the basis of all the studies performed on PPWs, the presented solutions have some limitations. The most important restriction is the non-generality of these solutions which cannot be used for a wide range of parameters. In fact, most of the proposed methods developed for estimating the pseudoskin are not valid for all the time periods of the test, especially for the early time period. On the other hand, in most studies, wellbore storage and skin factor have not been included in the solutions. To obtain a comprehensive analytical formula for different reservoirs, some researchers have recently developed several new analytical solutions for different types of the well-reservoir configurations [23-27]. As our main novelties, we present the objectives of this study as follows:

I. Deriving a general analytical solution that can be used to model the pressure transient behavior of a PPW completed in a reservoir with a diversity of structures.

II. Examining the validity of the presented technique by analyzing the responses of a PPW in a special case: homogenous reservoir with infinite radial extent.

The organization of this paper is as follows. In section 2, some basic concepts, which are required for the subsequent sections are introduced. In section 3, the analytical solution of the responses of a PPW is derived. Then, the analytical solution for the pressure behavior of a special case is given in section 4. Afterwards, the proposed solutions obtained in the previous sections are assessed and discussed in section 5. Finally, section 6 provides some concluding remarks.

2. Basic concepts

Before introducing the main results, some important preliminary concepts should be explained. These concepts are given in the following three subsections.

2.1. Source function

On the basis of the results obtained by Gringarten [28], if D_w and S_e are the source domain and boundary of the reservoir, respectively and M_w and M' are dummy points in the source and boundary, respectively, then the pressure drop at location M and time t, in the reservoir with initial pressure distribution $p_i(M)$, and a prescribed pressure or flux at the boundary S_e, can be written as [23]:

$$\Delta p(M,t)=\frac{1}{\phi c}\int_0^t\int_{D_w} q(M_w,\tau)G(M,M_w,t-\tau)dM_w d\tau$$
$$-\frac{k}{\phi\mu c}\int_0^t\int_{S_e}\left(G(M,M',t-\tau)\frac{\partial p(M',\tau)}{\partial n(M')}-\frac{\partial G(M,M',t-\tau)}{\partial n(M')}p(M',\tau)\right)dM'd\tau \quad (1)$$

where the pressure drop is defined as follows:

$$\Delta p(M,t)=\int_D p_i(M')G(M,M',t)dM'-p(M,t), \quad (2)$$

in which D denotes reservoir domain. For a reservoir with an initial uniform and constant pressure, $p(M,0)=p_i$, Eq. (2) can be expressed as:

$$\Delta p(M,t)=p_i-p(M,t). \quad (3)$$

The term $G(M,M_w,t)$ or $G(M,M',t)$ denotes the instantaneous Green's function. The expression $q(M_w,t)$ is the withdrawal (or injection) rate per unit volume ($M_w \in D_w$). Moreover, $\partial/\partial n(M')$ represents the directional derivative of the function p or G at a point M' in the outward direction of the boundary S_e, where the vector n is normal to the boundary S_e. For an infinite reservoir, the second term on the right-hand side of Eq. (1) approaches zero [23].

For simplicity, it can be assumed that the withdrawal flow rate is uniform over the source volume for a given reservoir, D. Since the second term on the right-hand side of Eq. (1) is not a function of the flow rate, it can be concluded that the given reservoir has a similar behavior to infinite reservoirs. Thus, Eq. (1) can be expressed as

$$\Delta p(M,t)=\frac{1}{\phi c}\int_0^t q(\tau)S(M,t-\tau)d\tau, \quad (4)$$

where $S(M,t)$ is the instantaneous source function that is defined as:

$$S(M,t)=\int_{D_w} G(M,M_w,t)dM_w. \quad (5)$$

The integral term on the right-hand side of Eq. (4) is called the continuous source function. For a source with constant fluid withdrawal rate, $q(t)=q$, Eq. (4) can be simplified to:

$$\Delta p(M,t)=\frac{q}{\phi c}\int_0^t S(M,t-\tau)d\tau. \quad (6)$$

It should be noted that Eq. (5) can be developed by means of the superposition principle.

2.2. Newman's method

The application of Newman's product method in the context of petroleum engineering can be expressed as follows: for a given reservoir, which can be considered as the intersection of one (and or two) dimensional reservoirs, the instantaneous Green's function equals to the multiplication of the instantaneous Green's functions for the one (and or two) dimensional reservoirs. In a similar approach, for a source that can be considered as the intersection of one (and or two) dimensional sources, the instantaneous source function is equal to the multiplication of the instantaneous source functions for the one (and or two) dimensional sources [29]. For example, the intersection of two perpendicular infinite plane sources constitutes an infinite line source. Thus, by making use of the Newman's product method, the instantaneous source function of an infinite line source is equal to the product of the instantaneous source functions of the corresponding plane sources:

$$S(x,y,t)=S(x,t)\cdot S(y,t). \tag{7}$$

And for another example, it can be considered a point source as the intersection of the three perpendicular infinite plane sources. Then, the instantaneous point source function can be obtained by means of the Newman's product method as:

$$S(x,y,z,t)=S(x,t)\cdot S(y,t)\cdot S(z,t). \tag{8}$$

If the point source is constituted by the intersection of an infinite plane source perpendicular to an infinite line source, then the corresponding instantaneous source function can be obtained as:

$$S(r,z,t)=S(r,t)\cdot S(z,t) \tag{9}$$

2.3. Gaver-Stehfest algorithm

The Gaver–Stehfest algorithm for the numerical inversion of Laplace transform was developed in the late 1960s [30]. This well-known method can be expressed as follows: the unknown pressure $P(r_D,t_D)$ for given r_D and t_D is obtained by using the Gaver-Stehfest algorithm to numerically invert the Laplace solutions $\overline{P}(r_D,s)$. The procedure is described by the following equations:

$$P(r_D,t_D)=\ln(2)t_D^{-1}\sum_{k=1}^{2n}a_k(n)\overline{P}(r_D,k\ln(2)t_D^{-1}), \qquad n\geq 1,\ t_D>0 \tag{10}$$

and the coefficients $a_k(n)$ are given by:

$$a_k(n)=\frac{(-1)^{n+k}}{n!}\sum_{j=[(k+1)/2]}^{\min(k,n)} j^{(n+1)}\binom{n}{j}\binom{2j}{j}\binom{j}{k-j}, \qquad n\geq 1,\ 1\leq k\leq 2n \tag{11}$$

where

$$\binom{m}{i}=\frac{m!}{i!(m-i)!}=\frac{m(m-1)\cdots(m-i+1)}{1\cdot 2\cdots i}, \qquad m\geq 0,\ 0\leq i\leq m \tag{12}$$

3. Analytical solution of the pressure transient behavior

The analytical solution of the pressure transient responses for a PPW flowing at a constant rate is now derived based on the source functions and the Newman's product method. Before presenting the solutions, some useful dimensionless variables are defined as follows:

Dimensionless pressure: $p_D=\dfrac{k_r h}{141.2q_w B\mu}\left(p_i-p(r,z,t)\right)$,

Dimensionless time: $t_D=\dfrac{0.0002637k_r t}{\phi\mu c_t r_w^2}$,

Dimensionless vertical distance: $z_D=\dfrac{z}{h}$, Dimensionless midpoint of the perforated interval: $z_{wD} = \frac{z_w}{h}$,

Dimensionless radius: $r_D = \frac{r}{r_w}$,

Dimensionless anisotropy group: $r_{Dw} = \frac{r_w}{h}\sqrt{\frac{k_z}{k_r}}$ and

Penetration ratio: $b = \frac{h_w}{h}$

The basic instantaneous source function for an infinite slab source in an infinite slab reservoir can be written as [24]:

$$S(z_D, t_D) = b\left[1 + \frac{4}{\pi b}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-n^2\pi^2 r_{Dw}^2 t_D)\sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D)\right], \tag{13}$$

where the top and bottom of the formation are impermeable. The instantaneous source functions for different boundary conditions can be found in [24]. It is worth to say that the following approach will work for a variety of the boundary conditions by choosing an appropriate source function. The total withdrawal rate from the well is:

$$q_w = q h_w, \tag{14}$$

where q is the flow rate per unit length of the source.

From the definition of the dimensionless pressure drop, it can be concluded that:

$$p_D(r_D, t_D) = \frac{1}{b} p_{Df}(r_D, t_D), \tag{15}$$

where the subscript f stands for a fully penetrating well (FPW). Since the dimensionless pressure drop is obtained by integrating the instantaneous source function with respect to time from 0 to t_D, taking the time derivative of Eq. (15) yields:

$$S(r_D, t_D) = \frac{1}{b} S(r_D, t_D)_f \tag{16}$$

where $S(r_D, t_D)$ represents the instantaneous source function for a PPW in r-direction. On the other hand, the dimensionless pressure drop for a FPW can be written as

$$p_{Df}(r_D, t_D) = \int_0^{t_D} S(r_D, \tau)_f \, d\tau \tag{17}$$

Differentiating both sides of Eq. (17), the instantaneous source function of a FPW can be obtained as follows:

$$S(r_D, t_D)_f = \frac{\partial p_{Df}(r_D, t_D)}{\partial t_D}. \tag{18}$$

Substituting Eq. (18) into Eq. (16) yields the instantaneous source function of a partially penetrating as:

$$S(r_D, t_D) = \frac{1}{b}\frac{\partial p_{Df}(r_D, t_D)}{\partial t_D}. \tag{19}$$

By using the Newman's product method, the instantaneous source function of the PPWs can be attained as:

$$S(r_D, z_D, t_D) = S(r_D, t_D).S(z_D, t_D)\,. \tag{20}$$

According to Eq. (20), the instantaneous source function for a well in partial penetration can be captured by combining Eq. (19) with Eq. (13); i.e,

$$\begin{aligned} S(r_D, z_D, t_D) = \frac{1}{b}\frac{\partial p_{Df}(r_D, t_D)}{\partial t_D}.b\Bigg(1 + \frac{4}{\pi b}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-n^2\pi^2 r_{Dw}^2 t_D) \\ \times \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D)\Bigg), \end{aligned} \tag{21}$$

which can be simplified to the following from:

$$S(r_D, z_D, t_D) = \frac{1}{b}\frac{\partial p_{Df}(r_D, t_D)}{\partial t_D}.b\big(1 + \eta(z_D, t_D)\big), \tag{22}$$

where

$$\eta(z_D, t_D) = \frac{4}{\pi b}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-n^2\pi^2 r_{Dw}^2 t_D)\sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D)\,. \tag{23}$$

Thus, the pressure response for a PPW produced at a constant rate can be written as:

$$p_D(r_D, z_D, t_D) = \int_0^{t_D} S(r_D, z_D, \tau)\, d\tau \tag{24}$$

By setting Eq. (21) into Eq. (24), the dimensionless pressure can be obtained as:

$$p_D(r_D, z_D, t_D) = p_{Df}(r_D, t_D) + \int_0^{t_D}\frac{\partial p_{Df}(r_D, \tau)}{\partial \tau}\eta(z_D, \tau)\, d\tau \tag{25}$$

The calculation of the pressure response of a PPW from Eq. (25) in an integration form is a computationally expensive process. This complexity arises from two reasons. First, the time derivative of $p_{Df}(r_D, t_D)$ must be evaluated. Second, the integration term must also be computed. These critical problems can be solved by transforming Eq. (25) to Laplace space, since the Laplace transformation can eliminate the derivative expression $\partial p_{Df}(r_D, \tau)/\partial \tau$ and the integral term. The Laplace transform of Eq. (25) yields

$$\overline{p}_D(r_D, z_D, s) = \overline{p}_{Df}(r_D, s) + \frac{1}{s}\sum_{i=1}^{N}\text{Residue}(\overline{\eta}(z_D, s_i))(s - s_i)\overline{p}_{Df}(r_D, s - s_i) \tag{26}$$

where

$$\overline{\eta}(z_D, s) = \frac{4}{\pi b}\sum_{n=1}^{\infty}\frac{1}{n(s + n^2\pi^2 r_{Dw}^2)}\sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D) \tag{27}$$

is the Laplace transform of Eq. (23), and s is the Laplace variable. The simple poles of Eq. (27) are located at $s = -n^2\pi^2 r_{Dw}^2$. So, the final result of the pressure drop without wellbore storage and skin can be written as:

$$\begin{aligned} \overline{p}_D(r_D, z_D, s) = \overline{p}_{Df}(r_D, s) + \frac{4}{\pi b s}\sum_{n=1}^{\infty}\frac{1}{n}(s + n^2\pi^2 r_{Dw}^2)\overline{p}_{Df}(r_D, s + n^2\pi^2 r_{Dw}^2) \\ \times \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D) \end{aligned} \tag{28}$$

The dimensionless pressure at the wellbore radius can be obtained by setting $r_D = 1$ in Eq. (28):

$$\bar{p}_D(1,z_D,s) = \bar{p}_{Df}(1,s) + \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(1,s+n^2\pi^2 r_{Dw}^2) \times \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D) \quad (29)$$

or

$$\bar{p}_D(z_D,s) = \bar{p}_{Df}(s) + \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(s+n^2\pi^2 r_{Dw}^2) \times \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D). \quad (30)$$

Partial completion of a well causes an additional pressure drop other than the pressure drop resulting from the full penetration, $\bar{p}_{Df}(s)$. This additional pressure drop is called geometrical skin or pseudoskin. The first term on the right-hand side of Eq. (30) is the dimensionless pressure drop for a FPW. So, it can be concluded that the pseudo skin pressure drop due to partial penetration is given by:

$$\bar{\sigma}(1,z_D,s) = \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(1,s+n^2\pi^2 r_{Dw}^2) \times \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D) \quad (31)$$

or

$$\bar{\sigma}(z_D,s) = \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(s+n^2\pi^2 r_{Dw}^2) \sin(\frac{n\pi b}{2})\cos(n\pi z_{wD})\cos(n\pi z_D) \quad (32)$$

It can be shown that the dimensionless wellbore pressure can be computed at $z_D = z_{wD} + r_{wD}$, where $r_{wD} = r_w/h$ [31-33]. Therefore, Eqs. (30) and (32) can be written as:

$$\bar{p}_D(s) = \bar{p}_{Df}(s) + \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(s+n^2\pi^2 r_{Dw}^2) \times \sin\left(\frac{n\pi b}{2}\right)\cos(n\pi z_{wD})\cos(n\pi(z_{wD}+r_{wD})) \quad (33)$$

and

$$\bar{\sigma}(s) = \frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}(s+n^2\pi^2 r_{Dw}^2)\bar{p}_{Df}(s+n^2\pi^2 r_{Dw}^2) \times \sin\left(\frac{n\pi b}{2}\right)\cos(n\pi z_{wD})\cos(n\pi(z_{wD}+r_{wD})) \quad (34)$$

with given penetration ratio, b, dimensionless anisotropy group, r_{Dw}, dimensionless midpoint of perforated interval, z_{wD}, and the calculated dimensionless vertical distance, z_D.

Dimensionless wellbore pressure with storage and skin for PPWs can be obtained by means of the superposition theorem which was given by van Everdingen and Hurst [34] and Agarwal et al. [35]:

$$p_{wD}(t_D) = \int_0^{t_D}\left[q_D(\tau)\frac{dp_D(t_D-\tau)}{d(t_D-\tau)}\right]d\tau + Sq_D(t_D), \quad (35)$$

where $p_{wD}(t_D)$ and $p_D(t_D)$ are the dimensionless wellbore pressure with wellbore storage and skin effects, and dimensionless sandface pressure for the constant-rate case without the wellbore storage and skin effects, respectively. Moreover,

$S =$ steady-state skin factor,

$p'_D(t_D) = {dp_D(t_D)}\big/{dt_D}$,

$C_D = \dfrac{5.6146C}{2\pi\phi\mu c_t h r_w^2}$, dimensionless wellbore storage,

$q_D(t_D) = 1 - C_D \dfrac{dp_{wD}(t_D)}{dt_D}$, dimensionless sandface rate

The Laplace transform of Eq. (35) yields

$$\bar{p}_{wD}(s) = \frac{s\bar{p}_D(s) + S}{s + C_D s^2 (s\bar{p}_D(s) + S)} \tag{36}$$

Replacing the $\bar{p}_D(s)$ term in Eq. (36) by the right-hand side of Eq. (33) yields the dimensionless pressure for PPWs with wellbore storage and skin. Dimensionless wellbore pressure values can be calculated from Eq. (36) by means of the Gaver-Stehfest numerical Laplace transform inversion algorithm [25].

4. Application of the proposed method

By applying the presented technique in section 3 to different types of reservoirs for analyzing the pressure behavior of PPWs, various mathematical models may be obtained. The simplest case is a homogenous and an infinite reservoir that is completed partially and is affected by wellbore storage and skin. Applying the proposed method to this case gives the following formulations.

The pressure response of a homogeneous, isotropic and infinite radial system with a FPW without wellbore storage and skin is given by:

$$\bar{p}_{Df}(r_D, s) = \frac{1}{s}\frac{K_0(r_D\sqrt{s})}{\sqrt{s}K_1(\sqrt{s})}. \tag{37}$$

Substituting Eq. (37) into Eq. (28) gives the pressure response of a PPW:

$$\begin{aligned}\bar{p}_D(r_D, z_D, s) = \frac{1}{s}\frac{K_0(r_D\sqrt{s})}{\sqrt{s}K_1(\sqrt{s})} + \frac{4}{\pi b s}\sum_{n=1}^{\infty}\frac{1}{n}\frac{K_0(r_D\sqrt{s + n^2\pi^2 r_{Dw}^2})}{\sqrt{s + n^2\pi^2 r_{Dw}^2}K_1(\sqrt{s + n^2\pi^2 r_{Dw}^2})}\\ \times\sin\left(\frac{n\pi b}{2}\right)\cos(n\pi z_{wD})\cos(n\pi z_D)\end{aligned}. \tag{38}$$

Equation (38) reduces to the following equation at the wellbore by considering $r_D = 1$ and $z_D = z_{wD} + r_{wD}$:

$$\begin{aligned}\bar{p}_D(s) = \frac{1}{s}\frac{K_0(\sqrt{s})}{\sqrt{s}K_1(\sqrt{s})} + \frac{4}{\pi b s}\sum_{n=1}^{\infty}\frac{1}{n}\frac{K_0(\sqrt{s + n^2\pi^2 r_{Dw}^2})}{\sqrt{s + n^2\pi^2 r_{Dw}^2}K_1(\sqrt{s + n^2\pi^2 r_{Dw}^2})}\\ \times\sin\left(\frac{n\pi b}{2}\right)\cos(n\pi z_{wD})\cos(n\pi(z_{wD} + r_{wD}))\end{aligned} \tag{39}$$

where the pseudoskin is

$$\bar{\sigma}(s)=\frac{4}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}\frac{K_0(\sqrt{s+n^2\pi^2r_{Dw}^2})}{\sqrt{s+n^2\pi^2r_{Dw}^2}K_1(\sqrt{s+n^2\pi^2r_{Dw}^2})}\sin\left(\frac{n\pi b}{2}\right) \times\cos(n\pi z_{wD})\cos(n\pi(z_{wD}+r_{wD})) \tag{40}$$

Substituting Eq. (39) into Eq. (36) yields the dimensionless wellbore pressure with wellbore storage and skin. Figure 1 shows the typical responses of a PPW in an infinite reservoir. For example, for a well that is perforated in the half-bottom interval where $z_{wD}=b/2$, Eqs. (39) and (40) can be simplified to:

$$\bar{p}_D(s)=\frac{1}{s}\frac{K_0(\sqrt{s})}{\sqrt{s}K_1(\sqrt{s})}+\frac{2}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}\frac{K_0(\sqrt{s+n^2\pi^2r_{Dw}^2})}{\sqrt{s+n^2\pi^2r_{Dw}^2}K_1(\sqrt{s+n^2\pi^2r_{Dw}^2})} \times\sin(n\pi b)\cos(n\pi(b/2+r_{wD})) \tag{41}$$

and

$$\bar{\sigma}(s)=\frac{2}{\pi bs}\sum_{n=1}^{\infty}\frac{1}{n}\frac{K_0(\sqrt{s+n^2\pi^2r_{Dw}^2})}{\sqrt{s+n^2\pi^2r_{Dw}^2}K_1(\sqrt{s+n^2\pi^2r_{Dw}^2})} \times\sin(n\pi b)\cos(n\pi(b/2+r_{wD})) \tag{42}$$

In addition, Eqs. (41) and (42) can be used when the half-top is perforated if the z-direction in the Cartesian coordinate is reversed.

5. Results and discussion

The pressure responses of a PPW in a reservoir with impermeable lower and upper boundaries have been presented in Figures 1, 2 and 3 with p_D versus t_D/C_D dimensionless variables. The well properties of the provided examples are summarized in Table 1. Figure 1 shows the pressure behavior of a PPW with different values of penetration ratio, given the values of dimensionless midpoint of perforated interval and dimensionless anisotropy group, where the value $b=1$ implies the FPW. Figure 2 depicts the effect of dimensionless midpoint of perforation interval with constant penetration ratio and dimensionless anisotropy group. The influence of dimensionless anisotropy group is analyzed by considering three different values, where the perforation ratio and the dimensionless midpoint of the perforation interval are given both by 0.5 (Figure 3). The dimensionless wellbore storage coefficient and skin have been assumed as $C_D=50$ and $S=2$, respectively.

Table 1. The well and reservoir properties of the study cases

	Dimensionless Anisotropy Group	Penetration Ratio	Dimensionless Midpoint of the Perforated Interval
Case 1	2.5×10^{-4}	$1, 0.8, 0.5, 0.3$	0.5
Case 2	2.5×10^{-4}	$b=0.5$	$0.25, 0.5, 0.75$
Case 3	$8\times10^{-4}, 2.5\times10^{-4}, 8\times10^{-5}$	$b=0.5$	0.5
Case 4	2.5×10^{-4}	$0.8, 0.5, 0.3$	0.5

As can be seen from Figures 1, 2 and 3, two radial flow regimes and one spherical flow between them can be observed in the responses for a PPW. At early times, is the initial radial flow over the perforated interval h_w, with Δp

proportional to $\log(\Delta t)$ and a first derivative stabilization. Then, the spherical flow can be observed with Δp proportional to $1/\sqrt{\Delta t}$ and a negative half unit slope straight line on the derivative log-log curve. The flow lines are established in both the horizontal and vertical directions, until the lower and upper boundaries are reached, which is the end of the spherical flow regime. Finally, the flow becomes radial in the entire formation thickness, with Δp proportional to $\log(\Delta t)$ and a second derivative stabilization.

Figure 4 presents the pseudoskin due to the partial penetration for three different penetration ratios, which indicates that the smaller b causes the higher pseudoskin. Based on the results shown in Figures 1 and 4, as the penetration ratio decreases, the pseudoskin and pressure drops increase. The results of Figure 3 show that with low dimensionless anisotropy group, the contribution of vertical flow is limited and the spherical flow is started later (r_w and h are assumed constant). When the completed interval is not centered within the entire formation thickness, the spherical flow ends when the closest bottom or top of the formation is reached. A hemi-spherical flow regime is then observed instead of the spherical flow, until the second boundary is reached (Figure 2). Furthermore, Figure 2 reveals that the pressure and its derivative results for the PPW are overlapped for the cases $b=\{0.8\}$ and $b=\{0.3\}$. It implies that if the well is symmetrically penetrated either in the half-top or half-bottom interval, the fluid flow in the reservoir will behave in a similar way.

The presented three different flow regimes can be analyzed to estimate the well and reservoir parameters. The permeability-thickness product for the perforated interval $k_r h_w$, and the skin, S can be obtained by analyzing the initial radial flow regime. The analysis of the spherical flow regime yields the permeability anisotropy. In addition, the permeability-thickness product of the reservoir $k_r h$, and the total skin S_t can be determined from the second radial flow regime.

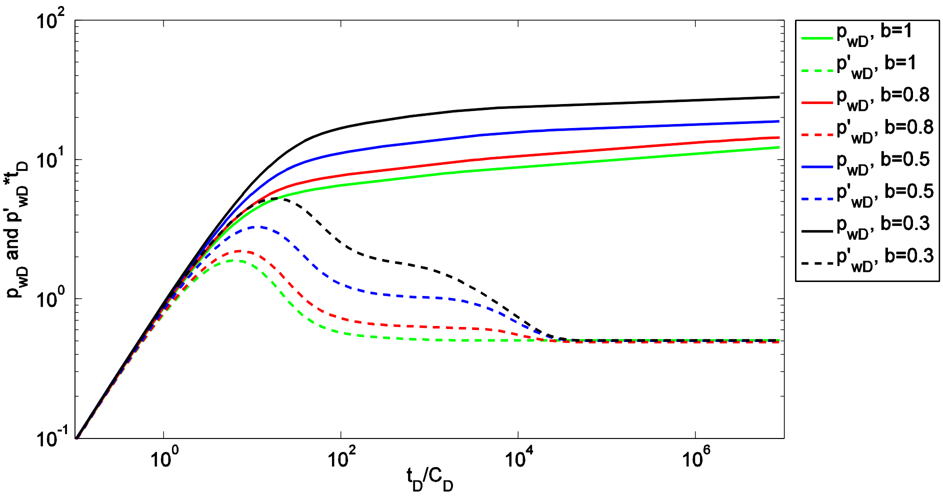

Figure 1. Comparison of the responses of fully and partially penetrating wells for different values of penetration ratio

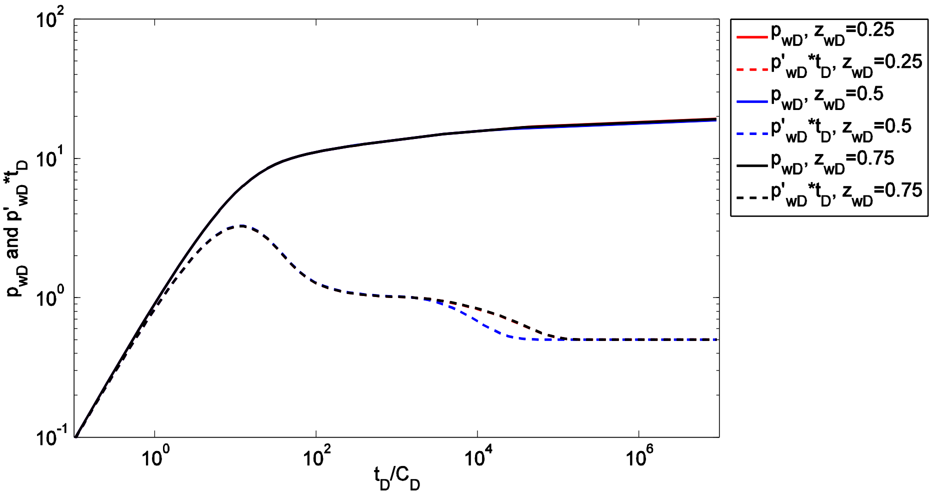

Figure 2. Effect of dimensionless midpoint of the perforated interval on dimensionless pressure drop and its logarithmic derivative

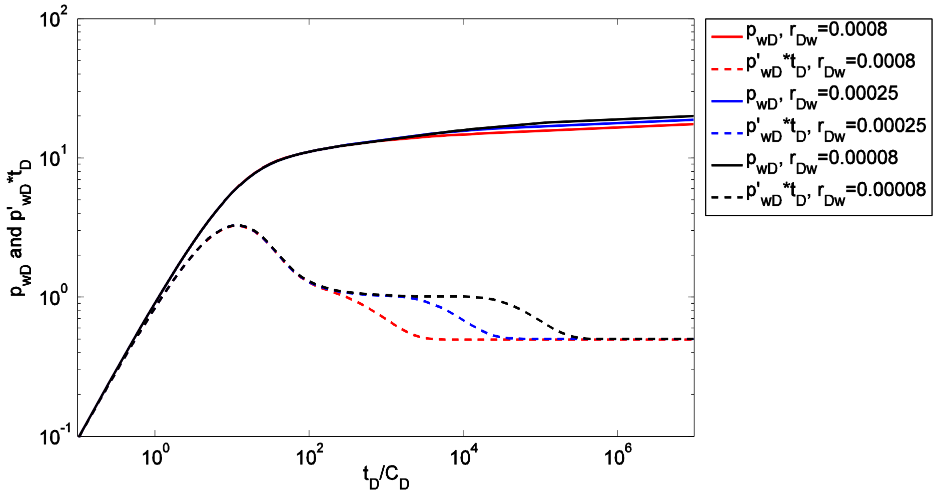

Figure 3. Effect of dimensionless anisotropy group on dimensionless pressure drop and its logarithmic derivative

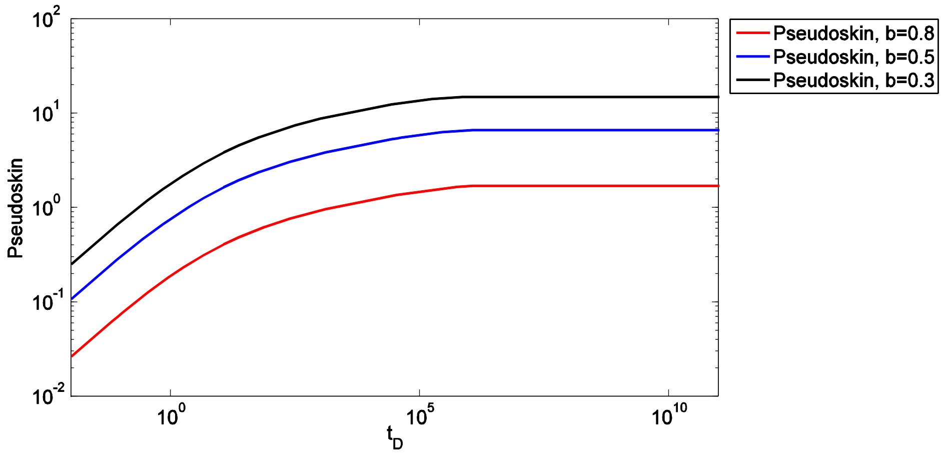

Figure 4. Pseudoskin for different values of penetration ratio when $z_{wD} = 0.5$ and $r_{Dw} = 0.00025$.

6. Conclusions

Based on the results presented and discussed earlier in this study, the following conclusions can be drawn:

1) A new analytical constant flow rate solution is derived based on the Green's function approach in Laplace space that describes the pressure behavior of partially penetrating (limited entry) wells at the wellbore when wellbore storage and skin effects are significant.
2) For any reservoir type and boundary condition, solutions of the PPWs can be obtained from the FPWs responses. Generality and simplicity of this technique can be used to define the responses for a PPW completed at any position in any region.
3) The pseudoskin pressure drop due to partial penetration increases as the spherical flow develops and then reaches its maximum value at large times. This is different from the mechanical skin, which is caused by formation damage in the drilling and completion operations.

Nomenclature

Symbols

b	penetration ratio [-]
c	isothermal compressibility factor [psi^{-1}]
C	wellbore storage constant [bbl/psi]
C_D	dimensionless wellbore storage constant [-]
h	thickness, producing interval [ft]
h_w	perforated interval [ft]
k	permeability [md]
k_r	horizontal (radial) permeability [md]
k_z	vertical permeability [md]
K_0	modified Bessel function of the second kind, zero order [-]
K_1	modified Bessel function of the second kind, first order [-]
p	pressure [psi]
p_D	dimensionless pressure [-]
p_{Df}	dimensionless pressure of fully penetrating vertical well [-]
p_i	initial pressure [psi]
p_{wD}	dimensionless wellbore pressure with wellbore storage and skin effects [psi]
p_{wf}	flowing BHP [psi]
q	flow rate per unit length of source [STB/D]
q_w	total withdrawal rate [STB/D]
r	distance from the center of wellbore [ft]
r_D	dimensionless radius [-]
r_{Dw}	dimensionless anisotropy group [-]
r_w	wellbore radius [ft]
S	skin factor [-]

t producing time [hours]
t_D dimensionless time [-]
u Laplace-transform variable with respect to t_D [-]
z distance along perforated interval [ft]
z_D dimensionless vertical distance [-]
z_w midpoint of perforated interval in the *z*-direction [ft]
z_{wD} dimensionless midpoint of perforated interval [-]
ϕ porosity of reservoir rock [-]
μ oil viscosity [cP]
σ pseudo skin pressure drop [-]
τ dummy integration variable [-]

Subscripts
D dimensionless
f fully penetrating
i initial
r horizontal (radial)
w bottom hole, well
z vertical

Superscript
$\_$ Laplace transform

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یک جواب جامع برای چاه‌های نفوذ جزیی با ساختارهای گوناگون مخزن

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چکیده

این مقاله جواب‌های تحلیلی و شبیه سازی شده برای رفتار فشار گذرای چاه‌های نفوذ جزیی را ارائه می‌دهد. با استفاده از روش ضرب نیومن، تابع‌های منبع آنی پایه، که پاسخ چاه‌های نفوذ جزیی را توصیف می‌کنند، تولید شده است. نتایج بر پایهٔ جواب‌های چاه‌های نفوذ کامل به دست آمده‌اند و این جواب‌ها در دامنهٔ لاپلاس ارائه شده‌اند. چندین مثال شبیه سازی شده جهت بررسی پاسخ یک چاه نفوذ جزیی و جهت تحلیل اثرات پارامترهای مختلف که نتایج نظری به دست آمده را تایید می‌کنند، ارائه شده است.

مشخصات مقاله

تاریخچهٔ مقاله:

دریافت: ۸ خرداد ۱۳۹۵

پذیرش نهایی: ۱۷ آبان ۱۳۹۵

کلمات کلیدی:

چاه نفوذ جزیی

تحلیل فشار گذرا

روش ضرب نیومن

چاه نفوذ کامل

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